NUMERICAL MODELING OF THE REBOUND OF AXISYMMETRIC RODS FROM A RIGID OBSTACLE

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The phenomena of propagation and interaction of loading and unloading waves, especially in one-dimensional and two-dimensional approximations, were investigated in a whole series of works (see, e.g., [1-6]). The analysis of the rebound process of rods at the present time is considered only in a one-dimensional approximation [7-10].

In this paper we numerically model the process of rebound of deformable rods of finite length from an absolutely rigid obstacle in a two-dimensional formulation.

We shall consider a class of problems connected with the collision of solid deformable bodies with different velocities in a two-dimensional formulation. Since body forces, heat conduction, and heat sources are absent, the equations of motion describing the stress–strain state of such a medium in Lagrangian coordinates have the form [11, 12]

$$\begin{gathered}
\frac{1}{\rho}\frac{\partial\rho}{\partial t}=-\left[\frac{\partial u}{\partial r}+\frac{\partial w}{\partial z}+\alpha\frac{u}{r}\right],\\
\rho\frac{\partial u}{\partial t}=\frac{\partial S_{rr}}{\partial r}+\frac{\partial S_{rz}}{\partial z}+\alpha\frac{S_{rr}-S_{\theta\theta}}{r}-\frac{\partial p}{\partial r},\\
\rho\frac{\partial w}{\partial t}=\frac{\partial S_{rz}}{\partial r}+\frac{\partial S_{zz}}{\partial z}+\alpha\frac{S_{rz}}{r}-\frac{\partial p}{\partial z},\\
\frac{\partial e}{\partial t}=\frac{p}{\rho^2}\frac{\partial\rho}{\partial t}+\frac{1}{\rho}\left[S_{rr}\frac{\partial u}{\partial r}+S_{zz}\frac{\partial w}{\partial z}+\alpha S_{\theta\theta}\frac{u}{r}+S_{rz}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)\right],\\
\frac{\partial r}{\partial t}=u,\quad \frac{\partial S_{rr}}{\partial t}=2\mu\left(\frac{\partial u}{\partial r}+\frac{1}{3\rho}\frac{\partial\rho}{\partial t}\right),\\
\frac{\partial z}{\partial t}=w,\quad \frac{\partial S_{zz}}{\partial t}=2\mu\left(\frac{\partial w}{\partial z}+\frac{1}{3\rho}\frac{\partial\rho}{\partial t}\right),\\
\frac{\partial S_{\theta\theta}}{\partial t}=2\mu\left(\alpha\frac{u}{r}+\frac{1}{3\rho}\frac{\partial\rho}{\partial t}\right),\quad \frac{\partial S_{rz}}{\partial t}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right),
\end{gathered}\tag{1}$$

where ρ is the density of the medium; u and w are the components of the velocity vector in projection onto the coordinate axes r and z respectively; t is the time; e is the specific internal energy. The stress tensor is represented in the form

$$\sigma_{ij}=-p\delta_{ij}+S_{ij}\quad(i=1,2,3,\ j=1,2,3),$$

where $p=\frac{1}{3}\sum_{1}^{3}\sigma_{ii}$; S_{ij} is the stress deviator with the components S_{rr}, S_{zz}, $S_{\theta\theta}$, S_{rz}; $\delta_{ij}=\begin{Bmatrix}1, & i=j\\ 0, & i\neq j\end{Bmatrix}$; μ is the shear modulus. It is assumed that the stresses for tension are positive and for compression are negative. The value $\alpha = 0$ corresponds to a plane strain state, while the value $\alpha = 1$ corresponds to an axisymmetric strain state.

For the closure of the system of equations (1) we use the plastic flow model, i.e., the second invariant of the deviator of the stress tensor in the region of plasticity satisfies the Mises yield condition

$$S_{rr}^2+S_{zz}^2+S_{\theta\theta}^2+2S_{rz}^2\leqslant\frac{2}{3}Y_0^2,\tag{2}$$

where Y_0 is the yield point determinable from results of uniaxial tension experiments. The equation of the state of the medium is represented in the form

$$P=a_1(\eta-1)+a_2(\eta-1)^2+a_3(\eta-1)^3+a_4\eta e,\tag{3}$$

where $\eta=\rho/\rho_0$; ρ_0 is the initial density; a_1, a_2, a_3, a_4 are constants. Further, $a_1=K$, $a_2=a_3=a_4=0$. In the presence of hardening the yield surface is altered in the loading process, which can be taken into account in the first approximation by means of a variable yield point [3].

Problem 1. We consider the problem of a longitudinal impact of a cylindrical rod of length L_0 and radius R_0 on an absolutely rigid obstacle with a velocity v_0 (Fig. 1). The mass of the cylinder is m.

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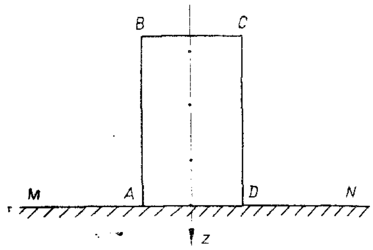

Fig. 1

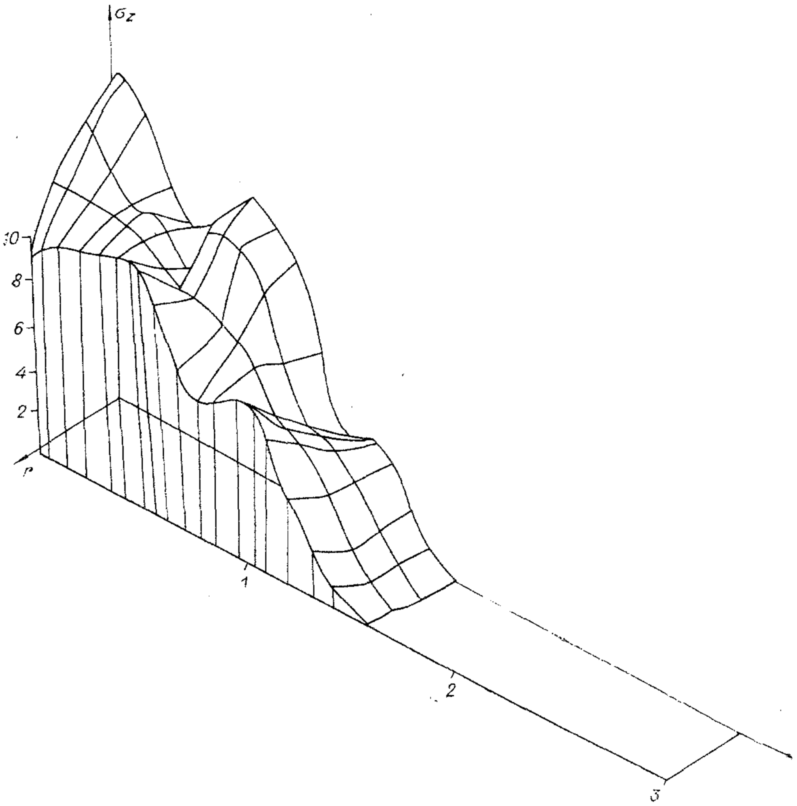

Fig. 2

The given physical problem can be formulated mathematically as follows: find functions ρ, u, w, σ_{ij} satisfying the system of equations (1)-(3), the initial conditions $w = v_0$, $\rho = \rho_0$, $\sigma_{ij} = 0$ in the region D (ABCD) and the boundary conditions $\sigma^{ij}n_j = 0$ on ABCD, $S_{rz} = 0$ and $w = 0$ on DA, where n_j are the components of the unit vector of the normal to the boundary ABCD. It should be noted that the boundary in the process of deformation changes, while the contact surface AD, consequently, will be variable. This is taken into account during the solution of the problem.

The solution of the problem is carried out by the Wilkins method [11]. The fundamental aim of the investigation at the same is to clarify the physics of the phenomenon of duration of the impact. This phenomenon only in one-dimensional formulation was investigated earlier in [3-9].

As the start of measurement of the duration of the impact we take the time instant when between the boundary of the body ABCD and the obstacle MN there appears at least one common point. This time instant is taken as $t = 0$.

If in the process of interaction of the body ABCD and the obstacle MN there arises a state in which the surfaces of the body and the obstacle have no common points, then this instant will be taken as the end of the impact duration. We take $t = t_*$ for it.

Definition. The phenomena which arise in the case of a collision of the body ABCD and the obstacle MN after the time interval t_* will be called a rebound. In the given case t_* is the time of contact or the time of duration of the impact.

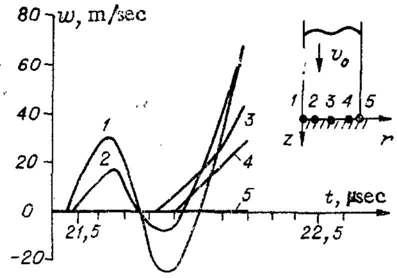

Fig. 3

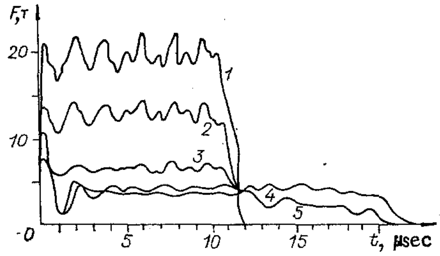

Fig. 4

The calculations carried out for different velocities of collision allowed us to compute the pattern of propagation of the elastoplastic waves and to understand the phenomena of rebound. The example of the complex stress state arising in a steel rod with parameters Y_0 = 0.012 mbar, R_0 = 0.3175 cm, L_0 = 3 cm, ρ = 7.87 g/cm^3, K = 1.7 mbar, μ = 0.8 mbar, v_0 = 75 m/sec for the time instant t_1 = 2.3 μsec is presented in Fig. 2, when the wave did not reach the surface BC. From the contact boundary DA at the beginning of the given process of collision there is formed a wave of compression of two-wave configuration, with the velocities of motion of these waves being substantially different. The first is an elastic wave, while the second is a plastic wave the amplitude and velocity of which substantially depends on the initial velocity of the impact. The presence of the transverse waves of the load, emerging from the side surfaces AB and DC, lower the intensity of the longitudinal waves, and therefore the calculated stress–strain state of the rod differs from the results of solution of problems obtained from the one-dimensional approximation. Having reached the free surface CB, the elastic wave of compression is reflected from it as an elastic wave of unloading and moves to meet the front of the plastic wave of the load. At the instant of their interaction, as follows from the solution of the local problem of break decomposition, the intensity of the plastic wave decreases with a jump, while toward CB there will again propagate the front of an elastic compression wave. Subsequently the process just described is being repeated as long as the system of elastic unloading waves does not remove completely the amplitude of the plastic compression wave. In each section r = const the process of wave interaction described above will be qualitatively analogous and will differ only by amplitudes and velocities of propagation. This leads to the circumstance that the unloading wave arrives at the surface MN in each section r = const at different time instants. Consequently, the contact of the points DA with MN is disturbed at the same time. This is particularly well seen in Fig. 3. By the numbers 1-5 we have shown the variation of the velocity of the points of the boundary DA, dependent on time. It is interesting to note that departing points of contact of DA can after a certain time again be joined with the rigid boundary MN. The nonsimultaneous separation of contact points of DA from MN in the given case, in contrast to the one-dimensional approximation, is substantially connected with its two-dimensionality.

For practical determination of the instant of rebound it is convenient to use the relation

$$F = \int_{S(t)} \sigma_z dS,$$

where S(t) is the boundary of the contact DA. The becoming zero of F corresponds to the instant of rebound. Calculations carried out for large values of the limit of elasticity Y_0, such that plasticity does not arise, show that the time of rebound is practically constant for rods of the same length in the case of different collision velocities (curves 1-3 in Fig. 4). The impact velocities respectively are 150, 100, and 50 m/sec. The curves 4 and 5 are calculations for Y_0 = 0.012 mbar and impact velocities 75 and 50 m/sec. The oscillating form of the relation is caused by the unloading waves from the free side surface. The time of rebound for curves 1-3 coincides with the time of arrival of the wave, reflected from the free end of the rod, moving with the rod velocity.

We consider the dependence of the dimensionless magnitude of the time of duration of the impact, t_*/t_0, on the velocity v_0 (Fig. 5, curve 1, t_0 is the time of duration of the impact in the case of an elastic collision). The growth of the duration of the impact (t_*/t_0) as the velocity increases has a substantially stepped character which, generally speaking, is qualitatively confirmed by results of experiments [8, 9]. The stepped form of this relation is explained in the first instance by the wave character of interaction of the plastic wave with the elastic unloading wave. The qualitative pattern of such a process was described above.

We consider the solution of Problem 1, but only in the one-dimensional formulation of the approximation of the stress state, when $\sigma_\theta = \sigma_r = 0$, the model of ideal plasticity. We shall show that the given problem in the class of continuous functions has no solution. For the proof we use the known solution of an analogous problem proposed in [1] in the framework of an elastic–plastic approximation of the Prandtl scheme. In the expressions obtained in [1, 2] we go to the limit with $C_1 \to 0$, where C_1 is the velocity of the plastic wave. Then the value of strains on the boundary of contact will be $\epsilon \to \infty$, which proves the statement.

Thus, it remains to be clarified as to why the solution in the two-dimensional formulation gives a qualitatively correct results. This fact is explained by the circumstance that in the solution of the problem 1 in the two-dimensional formulation, in comparison with the one-dimensional formulation, we take into account the variation of the cross-sectional

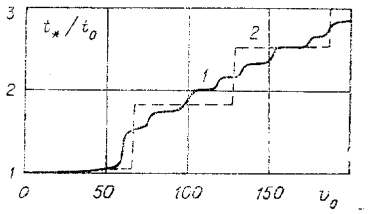

Fig. 5

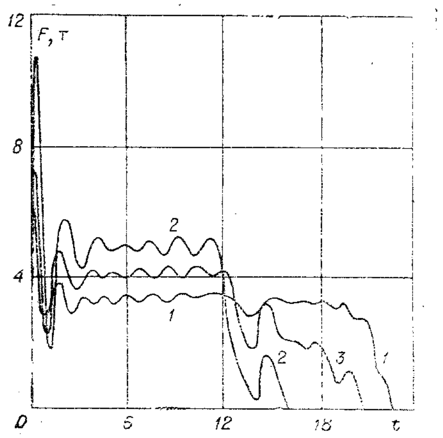

Fig. 6

area of the rod, especially in the plastic region during the passage of the elastic–plastic wave along the rod. The account in the given case of the variation of the area of section in the loading process for the behavior of the solution of the problem is qualitatively analogous to the account of hardening in the relation ($\sigma_z - \epsilon$) in the solution in the one-dimensional approximation.

We isolate in the initially cylindrical rod an elemental volume dV

$$dV = fdz,$$

obtained by sectioning the rod by two parallel planes the distance between which is dz. Here $f(z) \approx f(z + dz/2) \approx [f(z) + f(z + dz)]/2$. The equation of conservation of mass of this elemental volume has the form

$$\rho dV = \rho_0 dV_0,$$

where ρ_0 and V_0 are the initial values. The equations of motion in this case for a quasihomogeneous approximation in Lagrangian coordinates is written in the form

$$f\rho dw/dt = dT/dz, \ dz/dt = w, \ T = \sigma_z f(z).$$

The system of equations obtained in the elastic region is closed by Hooke's law written in the form

$$\frac{1}{v}\frac{\partial v}{\partial t} = (1 - 2\nu)\frac{\partial w}{\partial z}, \quad \frac{\partial \sigma_z}{\partial t} = \mathrm{E}\frac{\partial w}{\partial z}, \tag{4}$$

where $v = \rho_0/\rho$; $\nu = \lambda/2(\lambda + \mu)$; $\mathrm{E} = \mu(3\lambda + 2\mu)/(\lambda + \mu)$; λ and μ are the Lamé parameters. The relation (4) is obtained with the condition that $\sigma_r = \sigma_\theta = 0$, $\sigma_z \neq 0$, $\dot{\varepsilon}_r = -\nu\dot{\varepsilon}_z$, where the dots signify differentiation with respect to time.

In the region of plastic strain we have

$$\sigma_z = -p + S_z,$$
$$S_z = \pm(2/3)Y_0, \ p = \mp(1/3)Y_0.$$

Since in the plastic region the material of the body is incompressible, the volume remains constant, the fact which allows us to find the variation of the cross-sectional area f.

Thus, the given physical problem reduces to the following mathematical one: find functions ρ, w, σ_z, f, satisfying in the region $D_z(0 \leqslant t < \infty, \ -L_0 \leqslant z \leqslant 0)$ the system of equations

$$f\rho\frac{\partial w}{\partial t} = \frac{\partial T}{\partial z}, \quad \rho dV = \rho_0 dV_0, \quad T = \sigma_z f(z),$$
$$\frac{\partial \sigma_z}{\partial t} = \mathrm{E}\frac{\partial w}{\partial z}, \quad f = \frac{\partial V}{\partial z}, \quad \frac{1}{v}\frac{\partial v}{\partial t} = (1 - 2\nu)\frac{\partial w}{\partial z} \quad \text{for} \quad \sigma_z < Y_0; \tag{5}$$

$$f\rho\frac{\partial w}{\partial t} = \frac{\partial T}{\partial z}, \quad \rho dV = \rho_0 dV_0, \quad \frac{\partial \sigma_z}{\partial t} = \mathrm{E}\frac{\partial w}{\partial z},$$
$$T = \sigma_z f(z), \quad \sigma_z = \operatorname{sgn}(\sigma_z) Y_0 \quad \text{for} \quad \sigma_z \geqslant Y_0, \tag{6}$$
$$dV^* = \left[(1 - 2\nu)\frac{\sigma_z}{\mathrm{E}} + 1\right] dV_0, \quad f = \frac{dV^*}{dz};$$

the initial conditions $\rho = \rho_0$, $\sigma_z = 0$, $w = w_0$ in the region D_z; the boundary conditions

$$z = -L_0, \ \sigma_z = 0 \text{ for } (5), \ z = 0, \ w = 0 \text{ for } (6)$$

and the condition on the unknown moving boundary $-L_0 \leqslant z_1(t) \leqslant 0$

$$\sigma_z = Y_0, \ [v_1] = 0.$$

Here v_1 is the displacement; [ ] is the jump of the corresponding quantity.

The problem thus formulated was solved by a numerical method analogous to [11]. The results of the relations calculated, t_*/t_0, depending on the velocity of impact are shown in Fig. 5 by curve 2. We see that the given relation, just as the curve 1, is a clearly expressed stepped function. However, the quantitative results of these two solutions differ substantially.

Problem 2. We consider the problem of a longitudinal impact of truncated conical rods of length L_0 with radii R_0 and R_1 on an absolutely stiff obstacle with a velocity v_0 in the two-dimensional formulation.

The mathematical formulation of this problem is practically analogous to the Problem 1; therefore we do not present it here. The values of the forces of interaction F for two cones (L_0 = 3 cm, R_0 = 0.335 cm, R_1 = 0.3 cm – curve 1; L_0 = 3 cm, R_0 = 0.3 cm, R_1 = 0.335 cm – curve 2) and a cylinder (L_0 = 3 cm, R_0 = 0.3175 cm), the mass and kinetic energy of which are the same, are presented in Fig. 6 (v_0 = 75 m/sec). Even for small cone angles the time of rebound substantially differs from the time of rebound of the cylindrical rod. This phenomenon is connected with the process of accumulation of the elastic wave of loading and unloading as a result of the variation of the cross-sectional area and their interaction with the incident plastic wave. At the same time we should note that the impulse of action for the conical rod (L_0 = 3 cm, R_0 = 0.3 cm, R_1 = 0.335 cm) will be greater than for the cylindrical rod. This fact is interesting further by the circumstance that the same kinetic energy can be translated into plastic deformation in a different manner, dependent on the wave process being excited in the medium.

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